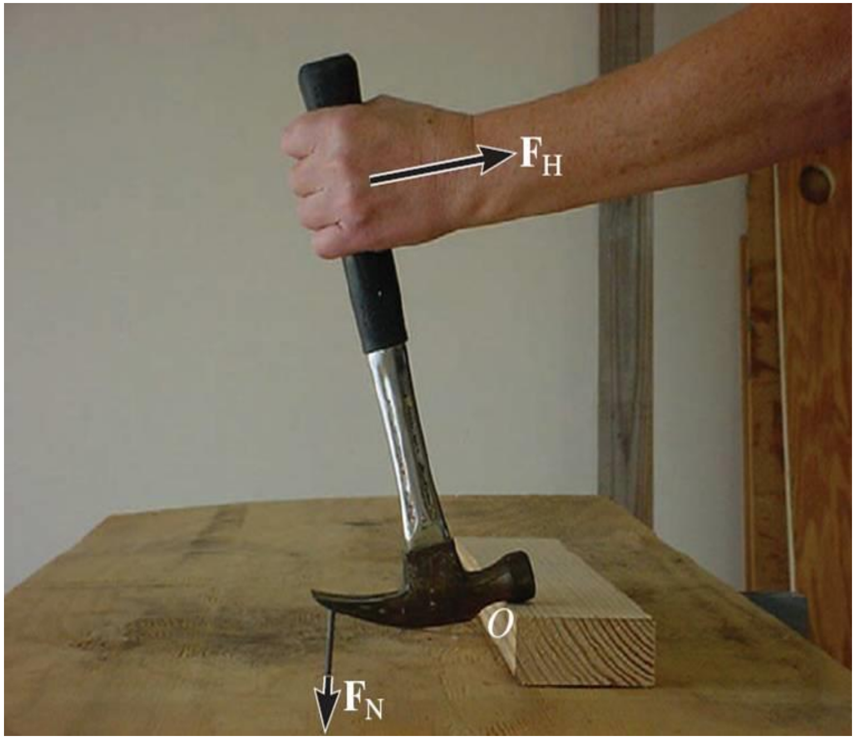
Chapter 4: Force System Resultants

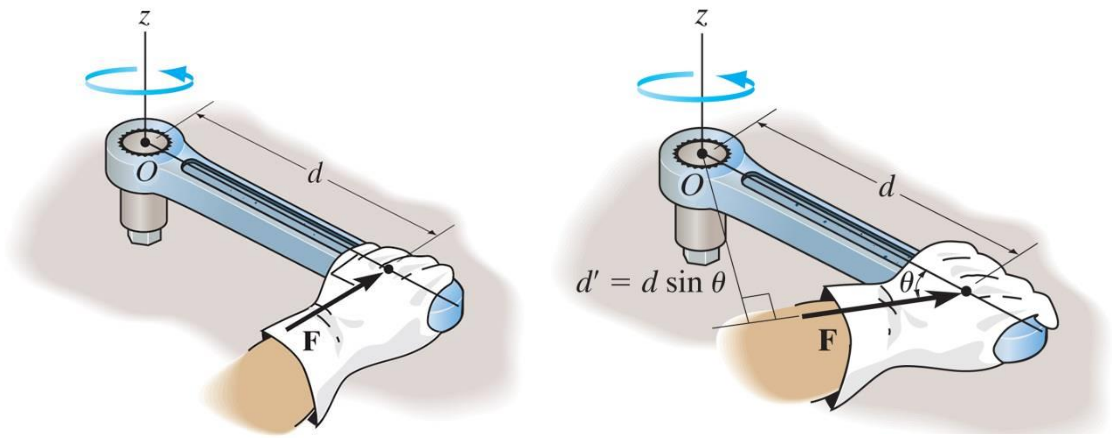
Applications

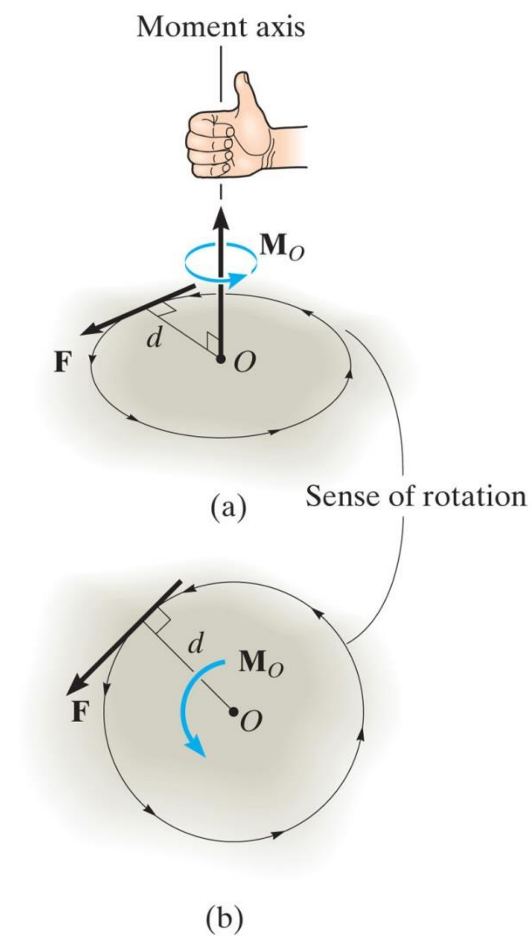


Carpenters often use a hammer in this way to pull a stubborn nail. Through what sort of action does the force F_H at the handle pull the nail? How can you mathematically model the effect of force F_H at point O?

Moment of a force – scalar formulation

The **moment of a force about a point** provides a measure of the **tendency for rotation** (sometimes called a torque).





Magnitude: In a 2-D case, the magnitude of the moment is $M_o = F\ d$

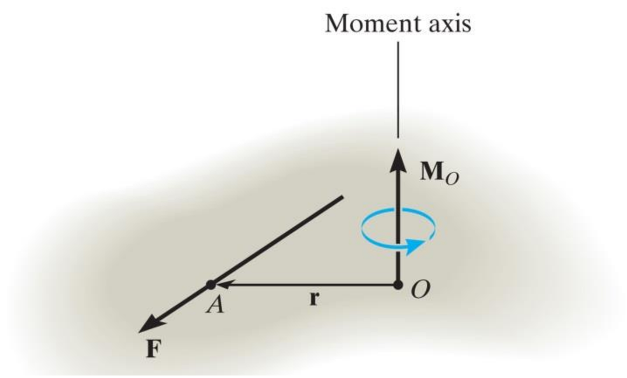
Direction: The moment is perpendicular to the plane that contains the force **F** and its moment arm d. The right-hand rule is used to define the sense.

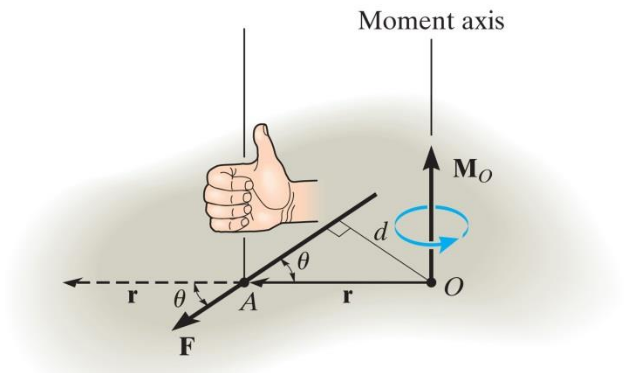
Moment of a force – vector formulation

The moment of a force **F** about point **O**, or actually about the moment axis passing through O and perpendicular to the plane containing **O** and **F**, can be expressed using the cross (vector) product, namely:

$$\boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F}$$

where r is the position vector directed from **O** to any point on the line of action of **F**.

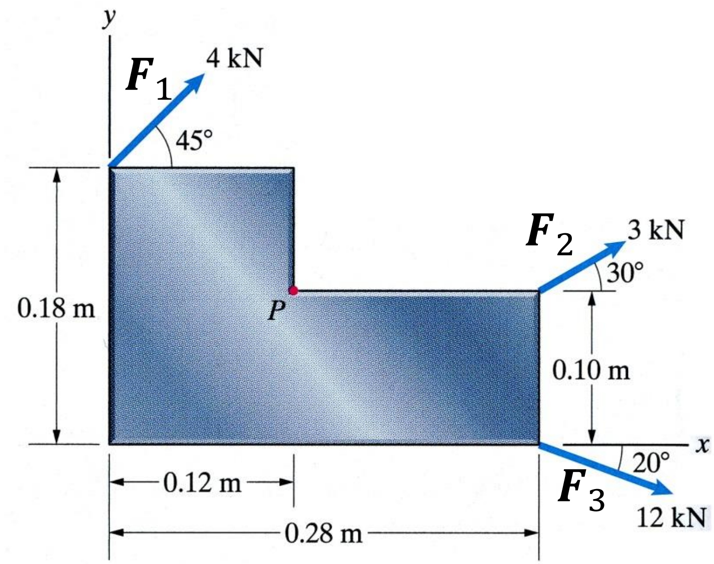


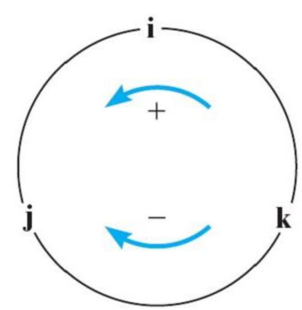


Magnitude:

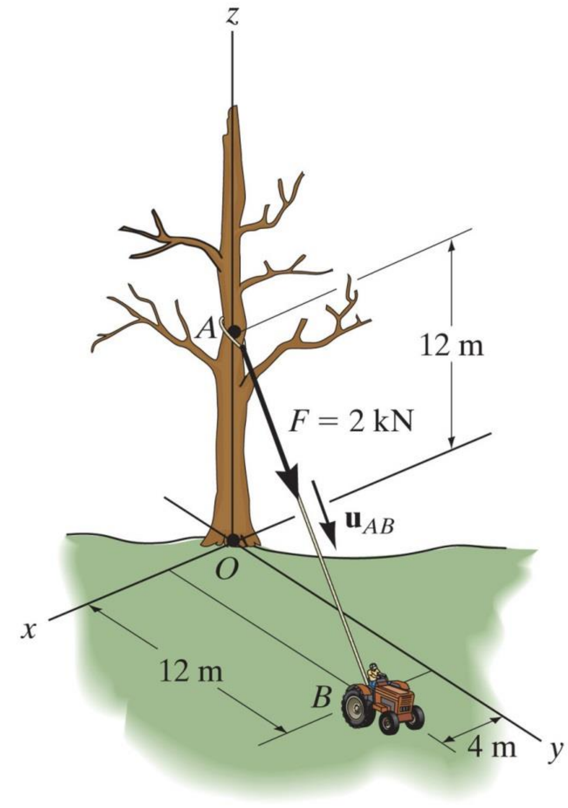
Direction: Defined by the right-hand rule

Three forces act on the plate. Determine the sum of the moments of the three forces about point P.





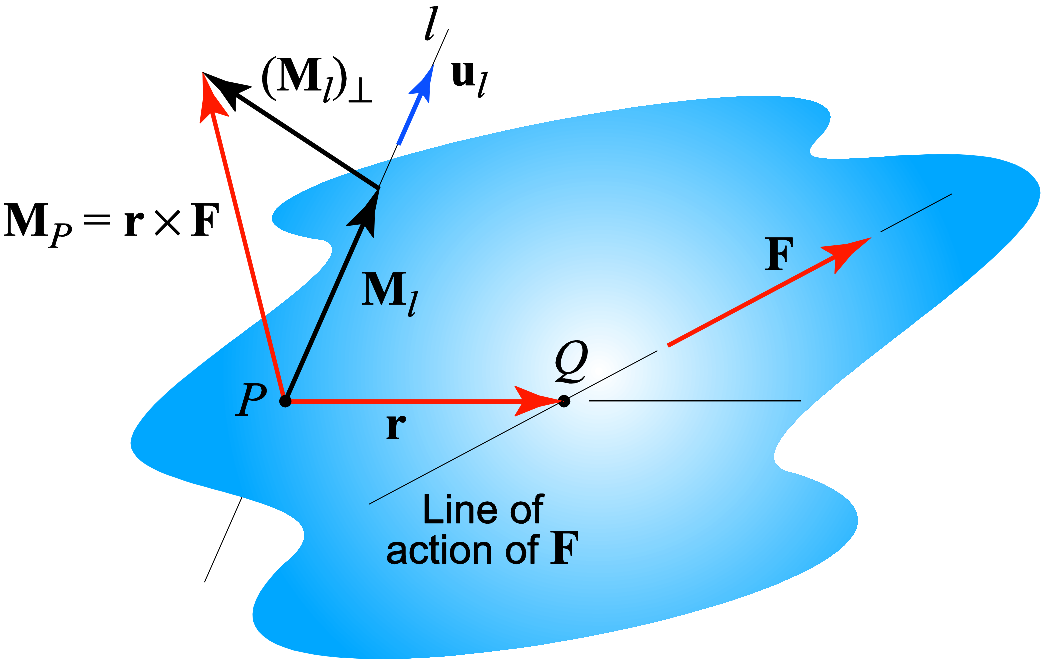
Determine the moment produced by the force **F** about point **O**.



Moment of a force about specified axis

Moments, like forces, can be resolved into components along and perpendicular to a given direction.

Let us assume we want to determine the moment of the force **F** about the line ***l***.



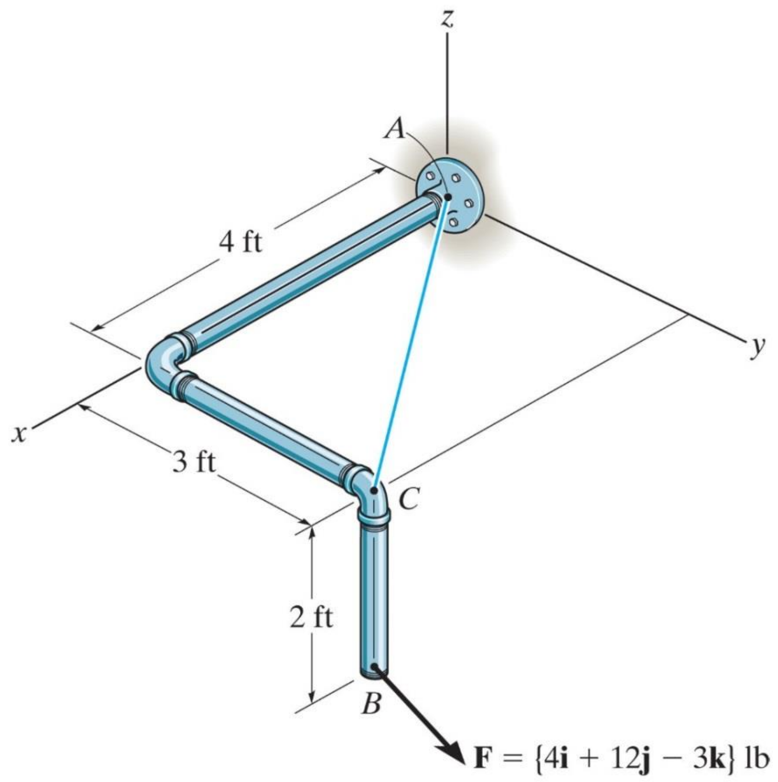
First we determine the moment of the force **F** about any point on the line ***l*** (e.g. point P).

Then we obtain the projection of $\boldsymbol{M}_P$ along the line ***l*** using the dot product.

The moment vector $\boldsymbol{M}_l$ can now be expressed as:

And the perpendicular component is:

Determine the moment of the force F about the an axis extending between A and C.



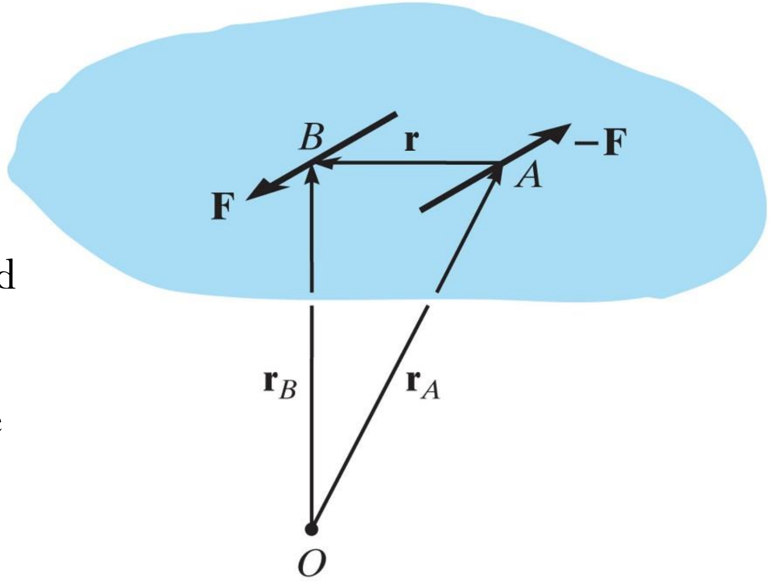
Moment of a couple

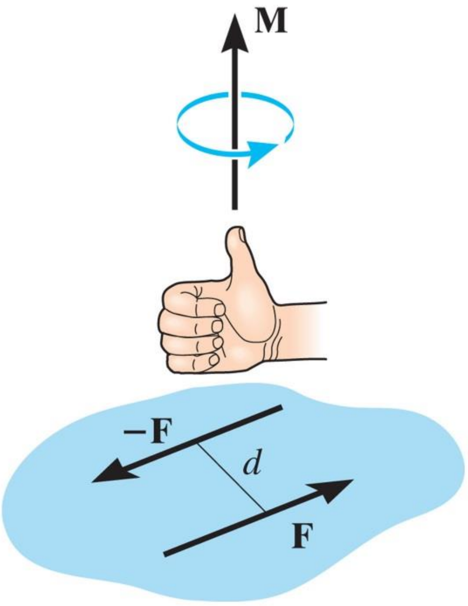
A **couple** is defined as two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d.

Since the resultant force is zero, the only effect of a couple is to produce an actual rotation, or if no movement is possible, there is a tendency of rotation in a specified direction.

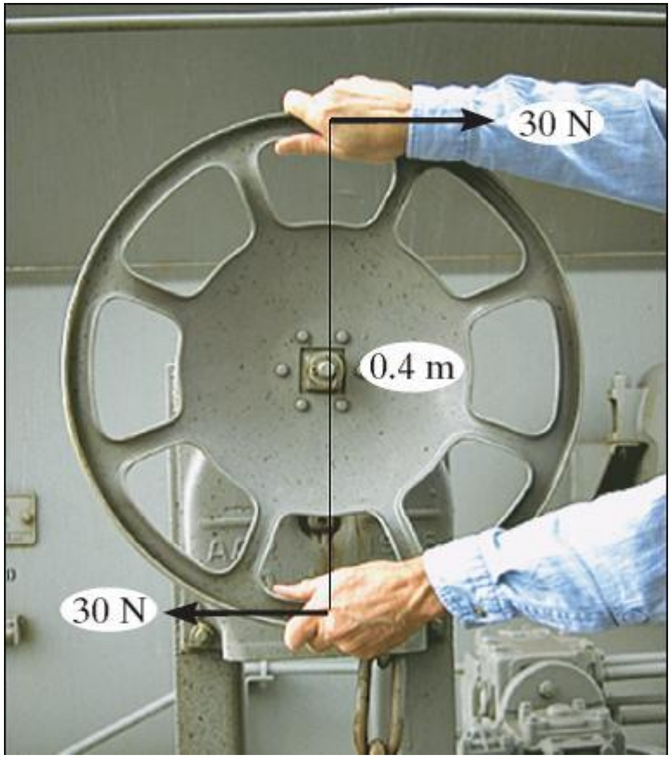
The moment produced by a couple is called **couple moment**.

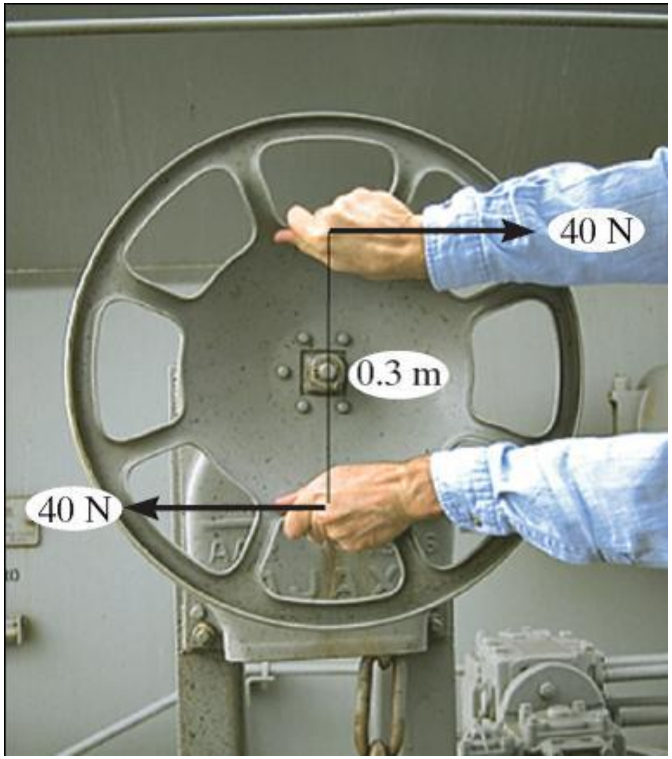
Let's determine the sum of the moments of both couple forces about **any** arbitrary point:





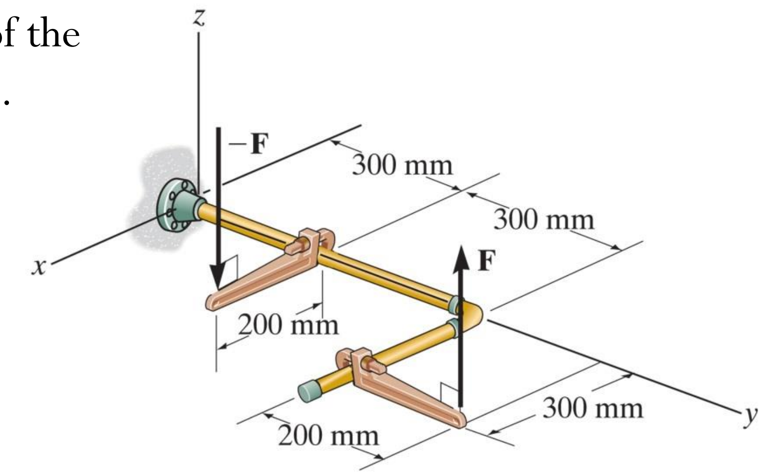
Couple moment is a **free vector**, i.e. is **independent** of the choice of O!



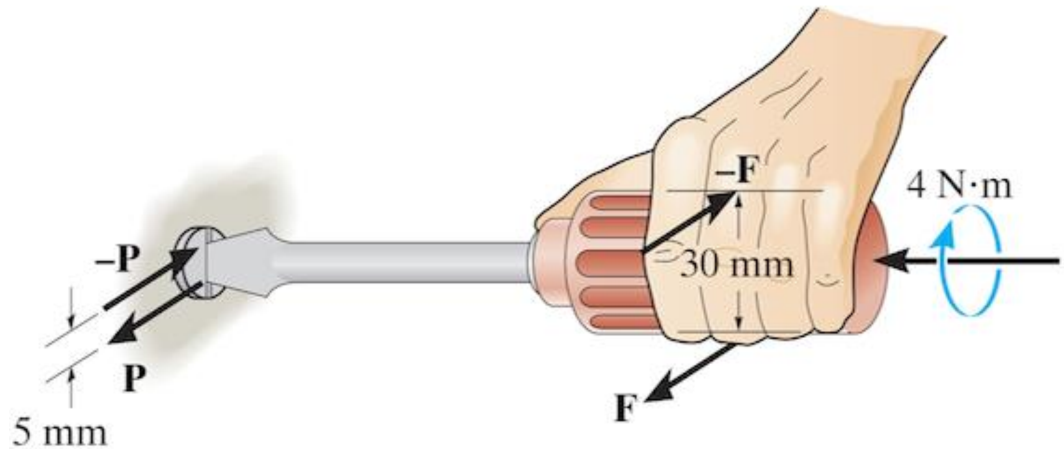


A torque or moment of 12 N·m is required to rotate the wheel. Why does one of the two grips of the wheel above require less force to rotate the wheel?

Determine the magnitude and coordinate direction angles of the couple moment. The pipe line assembly lies in the x-y plane. Assume F = 80 N.



A twist of 4 N·m is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple force F exerted on the handle and P exerted on the blade.



Equipollent (or equivalent) force systems

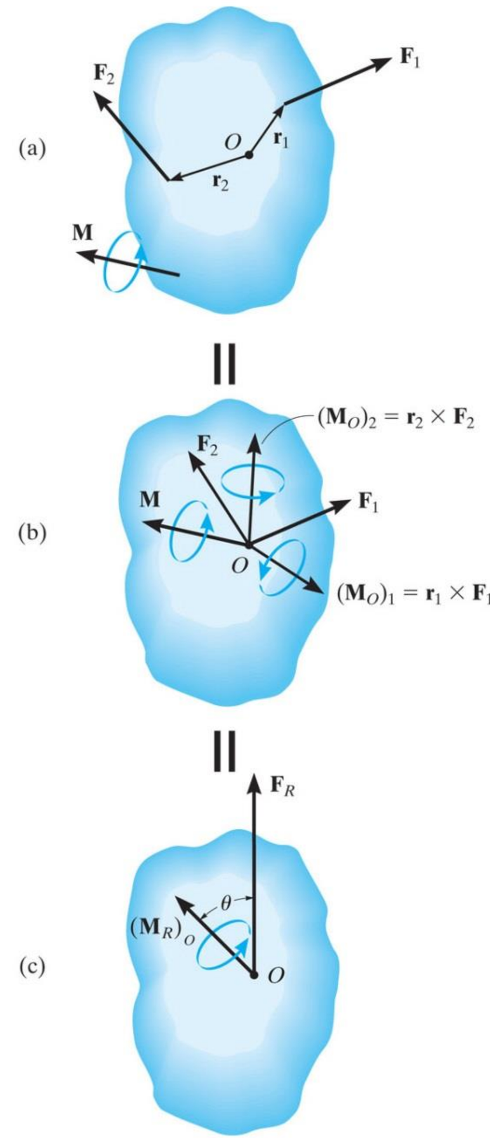
A force **system** is a collection of **forces** and **couples** applied to a body.

Two force systems are said to be **equipollent** (or equivalent) if they have the **same resultant force** AND the **same resultant moment** with respect to any point *P*.

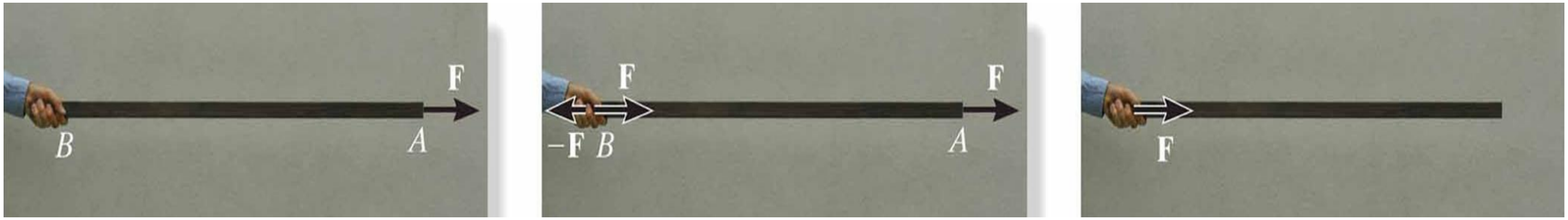
Reducing a force system to a single resultant force $\boldsymbol{F}_R$ and a single resultant couple moment $(\boldsymbol{M}_R)_O$:

$$\boldsymbol{F}_R = \sum \boldsymbol{F}$$

$$(\boldsymbol{M}_R)_O = \sum \boldsymbol{M}_O + \sum \boldsymbol{M}$$



Moving a force on its line of action



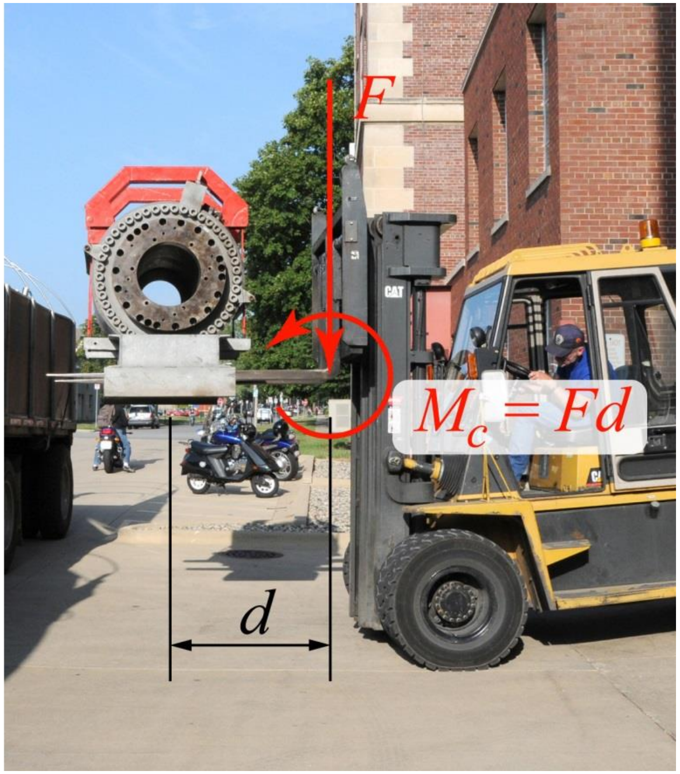
Moving a force from A to B, when both points are on the vector's line of action, does not change the **external effect**.

Hence, a force vector is called a **sliding vector**.

However, the **internal effect** of the force on the body does depend on where the force is applied.

Moving a force off of its line of action

Force system I



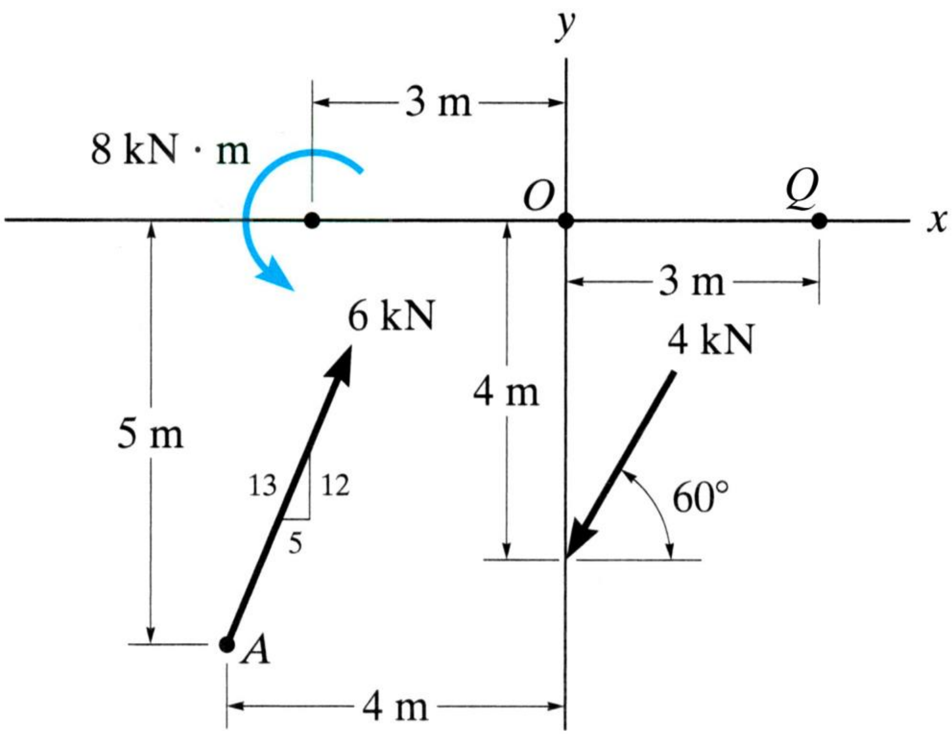
Force system II

The two force systems are equipollent since the resultant force is the same in both systems, and the resultant moment with respect to any point P is the same in both systems.

So moving a force off its line of action means you have to "add" a new couple. Since this new couple moment is a **free vector**, it can be applied at any point on the body.

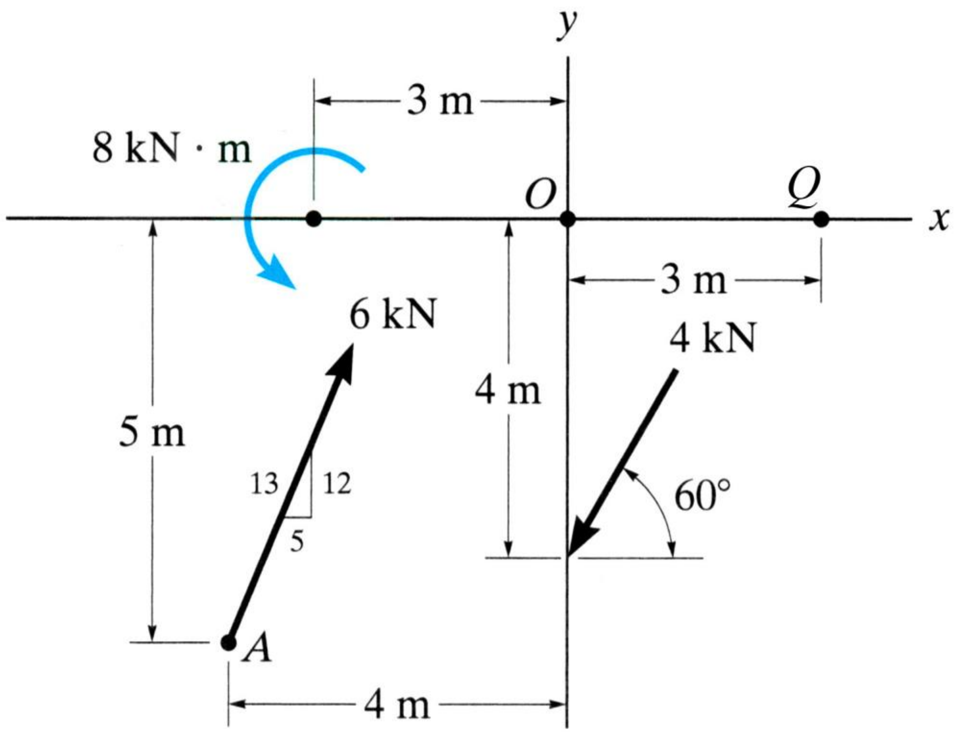
Problem

Replace the force and couple system by an equipollent force and couple moment at point *Q*.

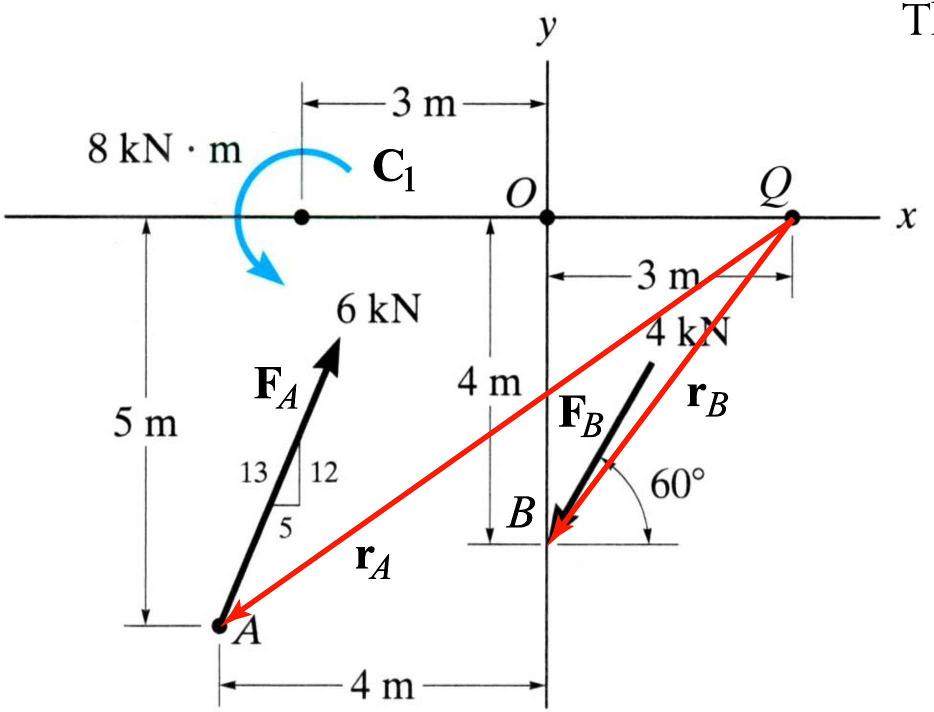


Solution

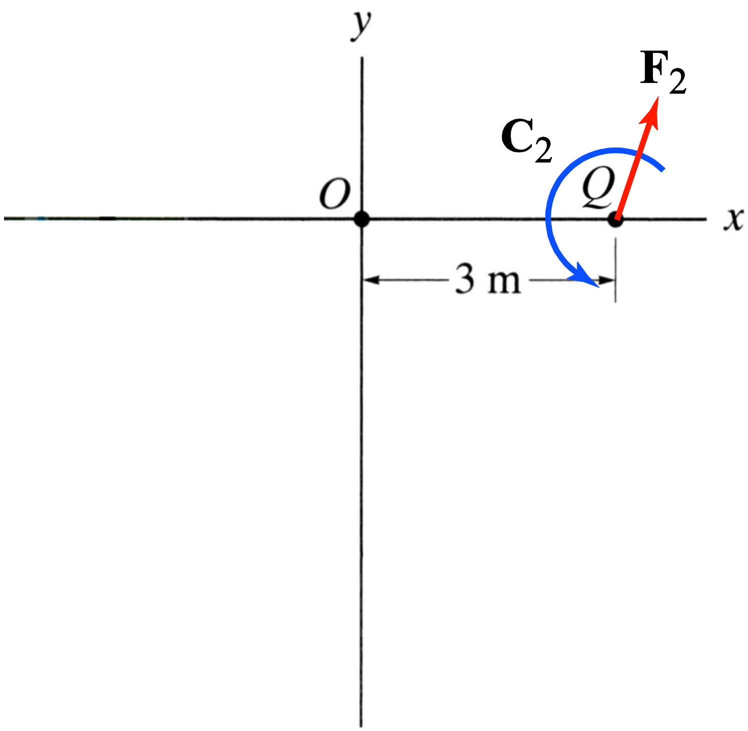
For equipollence, $(\Sigma \mathbf{F})_1 = (\Sigma \mathbf{F})_2$ and $(\Sigma \mathbf{M}_P)_1 = (\Sigma \mathbf{M}_P)_2$ with respect to *any* point P is required. Either point O or point Q would be fine.



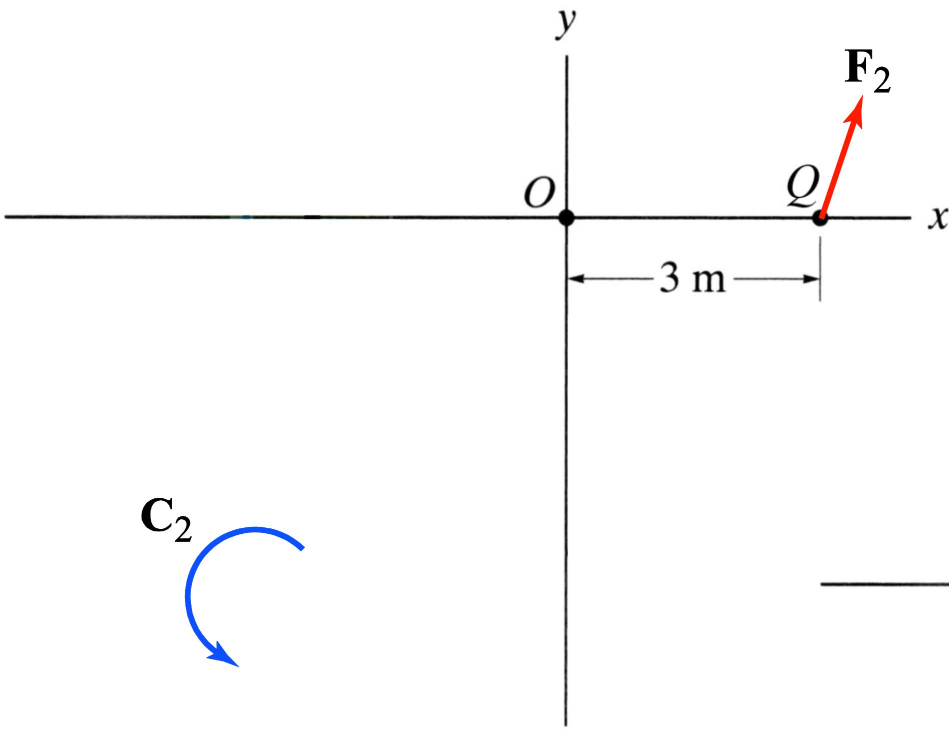
The simplest point is perhaps Q, since $(\Sigma \mathbf{F})_2$ will have no moment with respect to point Q.



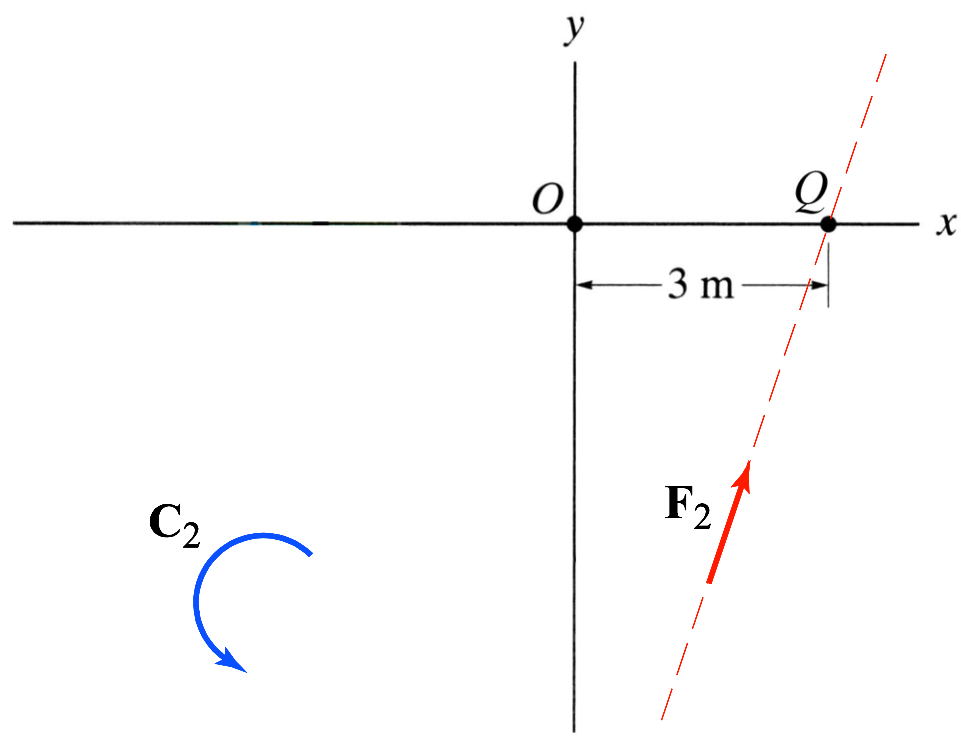
Then for force system 1, we have:



Now consider force system 2, consisting *only* of a force at Q and a couple:



Note that the couple $\boldsymbol{C}_2$ can be applied anywhere.



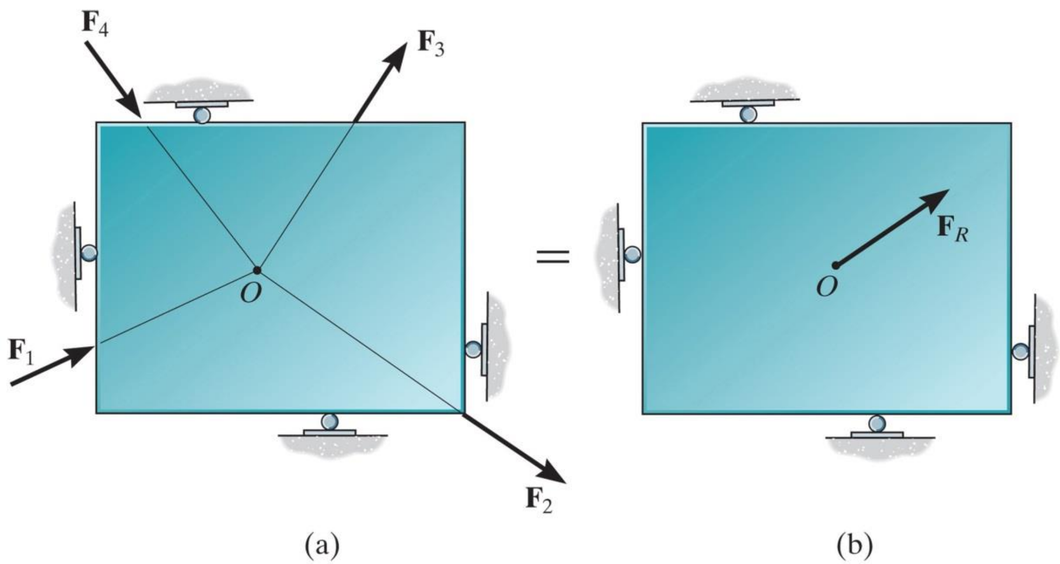
Also, the force $\boldsymbol{F}_2$ can be applied anywhere along its line of action

Special cases of equivalent systems

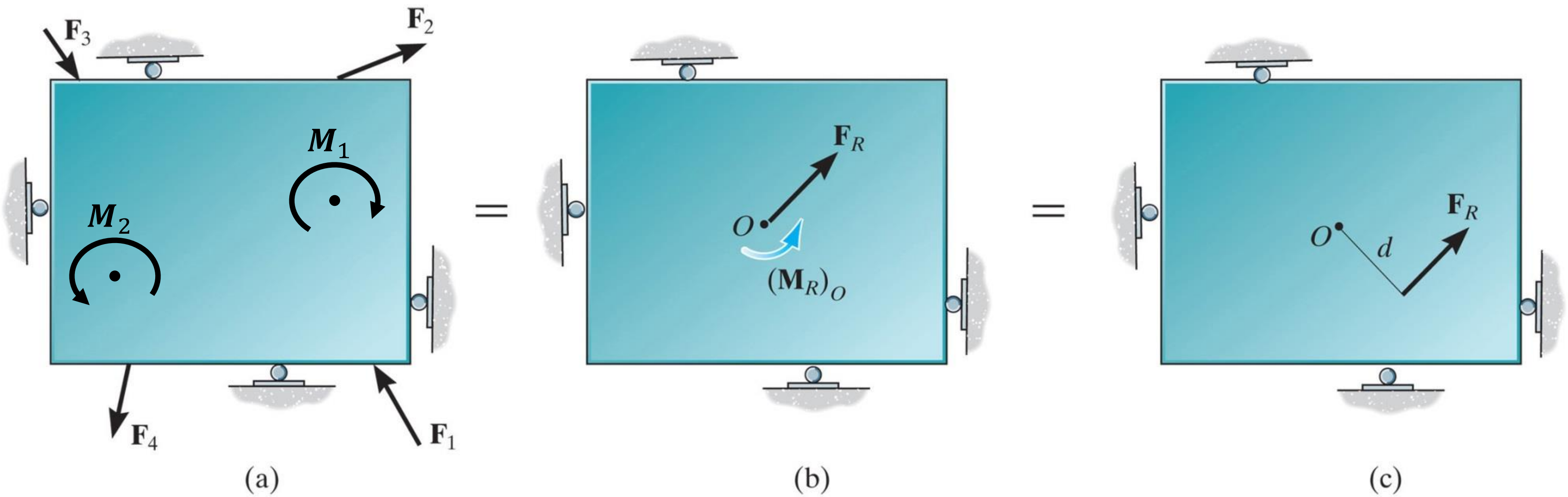
If $\boldsymbol{F}_R \perp (\boldsymbol{M}_R)_o$ and $\boldsymbol{F}_R \neq 0$, then an equivalent system consisting of **only** a single force can always be found. There are three possibilities:

Concurrent Force System

The lines of action of all the forces intersects at a common point O.



Coplanar Force System



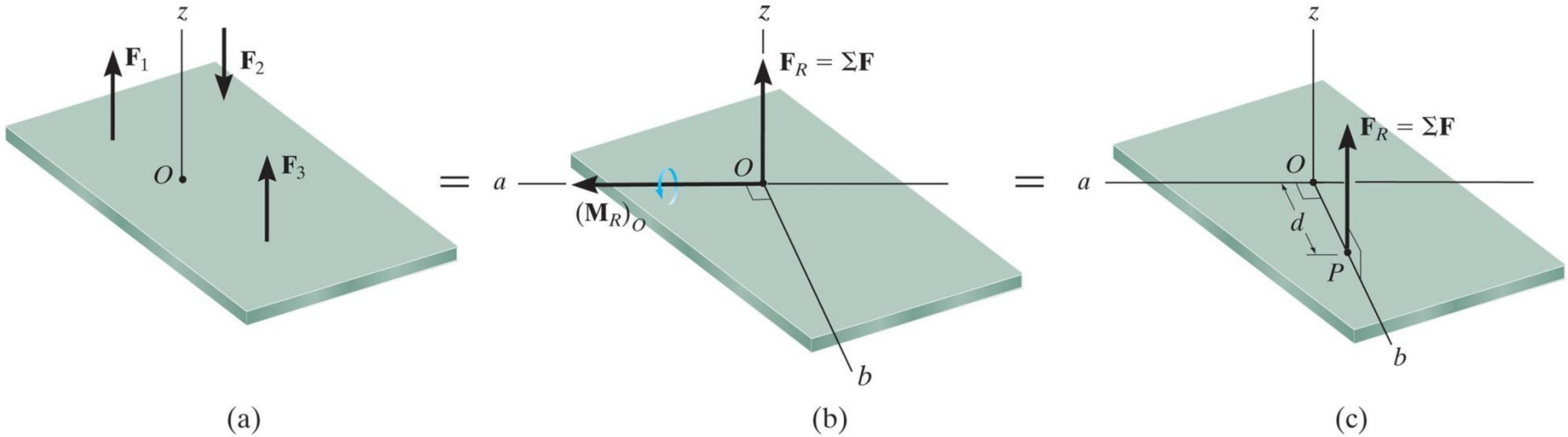
The lines of action of all the forces lie in the same plane, and so the resultant $\boldsymbol{F}_R$ also lies in the same plane.

The resultant moment $(\boldsymbol{M}_R)_o$ about any point O is perpendicular to the resultant force $\boldsymbol{F}_R$.

The resultant moment can be replaced by moving the resultant force $\boldsymbol{F}_R$ a moment arm distance "d" away from point O such that $\boldsymbol{F}_R$ produces the same moment $(\boldsymbol{M}_R)_o$ about the point O, i.e.

$$d = (\boldsymbol{M}_R)_o / \boldsymbol{F}_R$$

Parallel Force System



All forces are parallel to the z-axis.

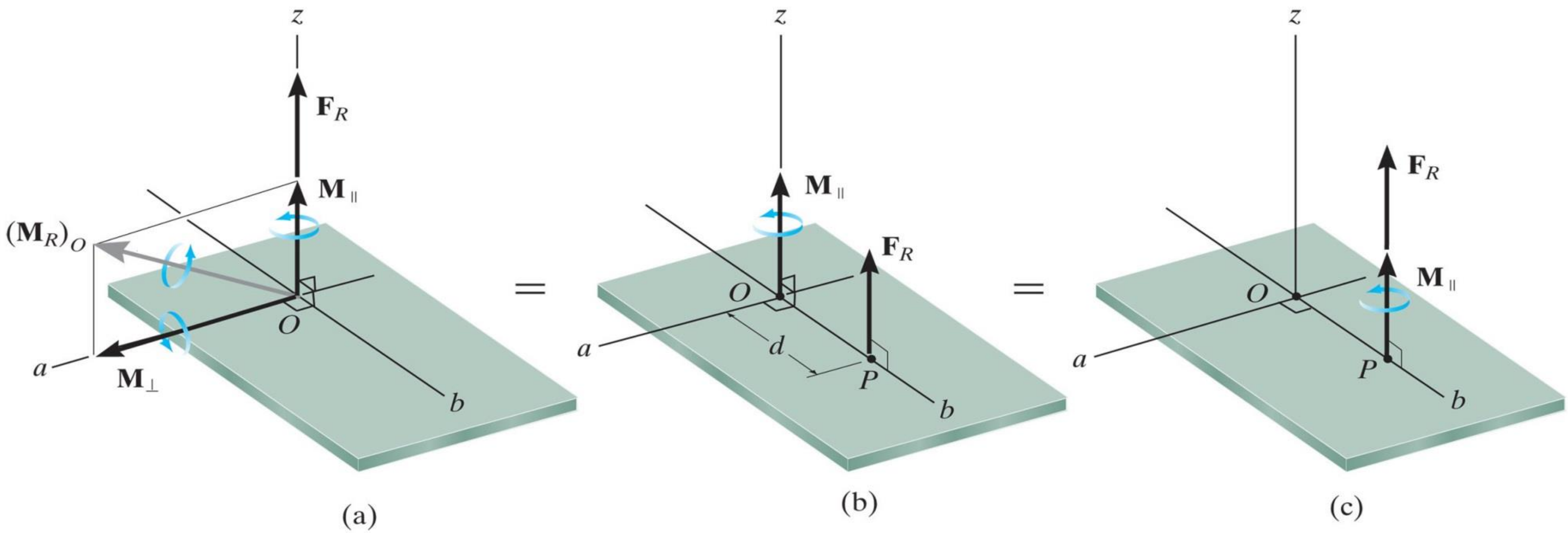
The resultant force $\boldsymbol{F}_R$ at a point O must also be parallel to the z-axis. Therefore, the resultant couple moment $(\boldsymbol{M}_R)_o$ is perpendicular to the resultant force $\boldsymbol{F}_R$ and lies in the plane x-y along the moment axis a.

The resultant moment can be replaced by having the resultant force $\boldsymbol{F}_R$ acting through point P, located on axis b perpendicular to axis a. The distance "d" along this axis from point O requires:

$$d = (\boldsymbol{M}_R)_o / \boldsymbol{F}_R$$

Reduction to a wrench

In general, a 3D force and couple moment system will have an equivalent resultant force $\boldsymbol{F}_R$ acting at point O and a resultant couple moment $(\boldsymbol{M}_R)_o$ that are NOT perpendicular to one another.



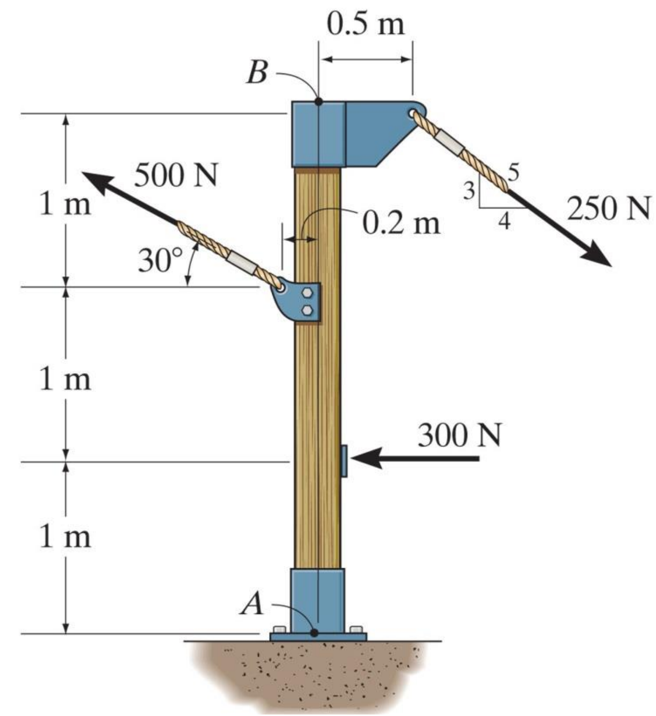
The resultant couple moment $(\boldsymbol{M}_R)_o$ can be resolved into components parallel and perpendicular to the line of action of the resultant force $\boldsymbol{F}_R$

The perpendicular component $\boldsymbol{M}_{\perp}$ can be replaced by moving $\boldsymbol{F}_R$ to point P, a distance "d" from point O along the b axis.

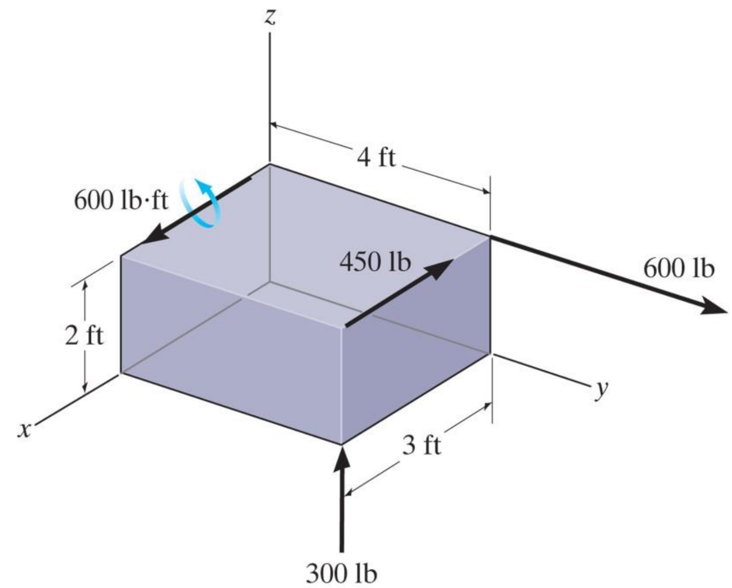
$$d = \boldsymbol{M}_{\perp}/\boldsymbol{F}_R$$

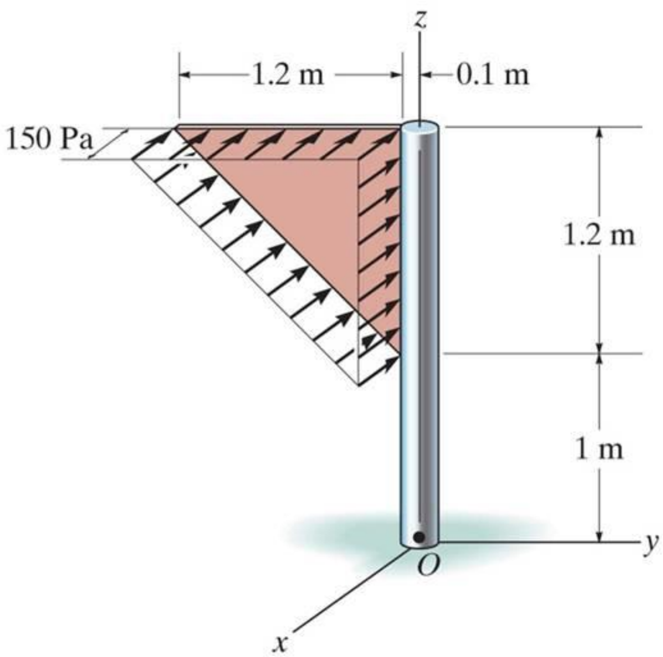
Because $\boldsymbol{M}_{\parallel}$ is a free vector, it can be moved to point P. This combination of a resultant force $\boldsymbol{F}_R$ and a collinear couple moment $\boldsymbol{M}_{\parallel}$ will tend to translate and rotate the body about its axis and is referred as **wrench** or **screw**.

Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post AB measured from point B.



Replace the force and the couple moment system acting on the rectangular block by a wrench. Specify the magnitude of the force and couple moment of the wrench and where its line of action intersects the x-y plane

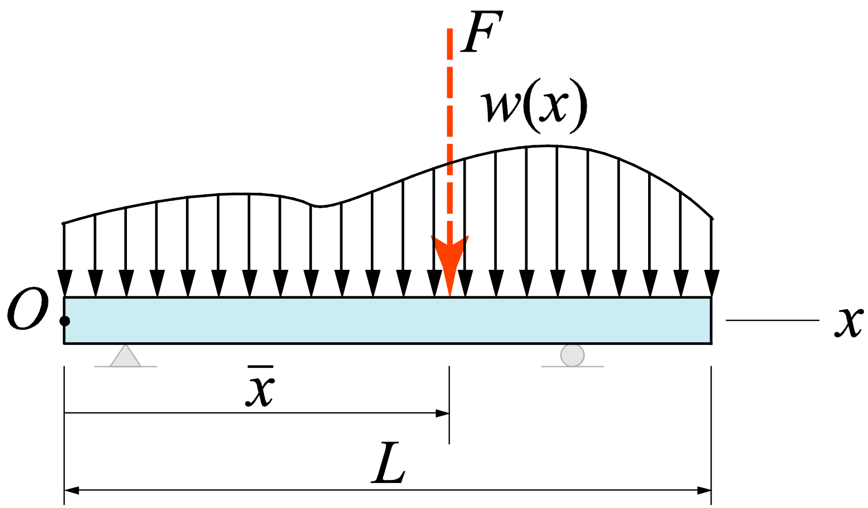




The lumber places a distributed load (due to the weight of the wood) on the beams. To analyze the load's effect on the steel beams, it is often helpful to reduce this distributed load to a single force. How would you do this?

To be able to design the joint between the sign and the sign post, we need to determine a single equivalent resultant force and its location.

Reduction to a simple distributed load



In structural analysis, we often are presented with a **distributed load** $w(x)$ (force/unit length) and we need to find the equivalent loading F.

Example of such forces are winds, fluids, or the weight of items on the body's surface.

By equipollence, we require that $\sum \boldsymbol{F}$ be the same in both systems, i.e.,

$$F = \int_0^L w(x)\, dx = A$$

and $\sum \boldsymbol{M}_P$ with respect to any point P be the same in both systems, i.e.,

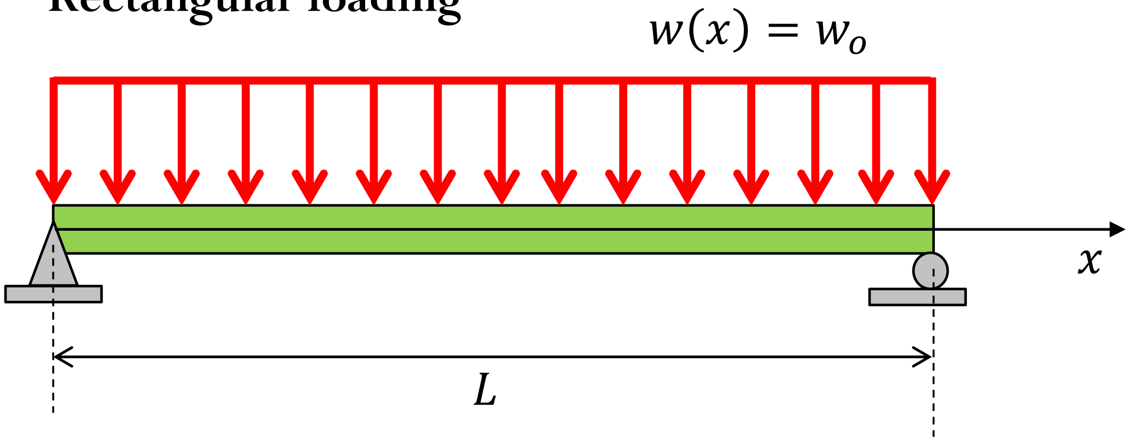
$$\int_0^L w(x)\, x\, dx = \bar{x}\, F$$

Combining both equations gives:

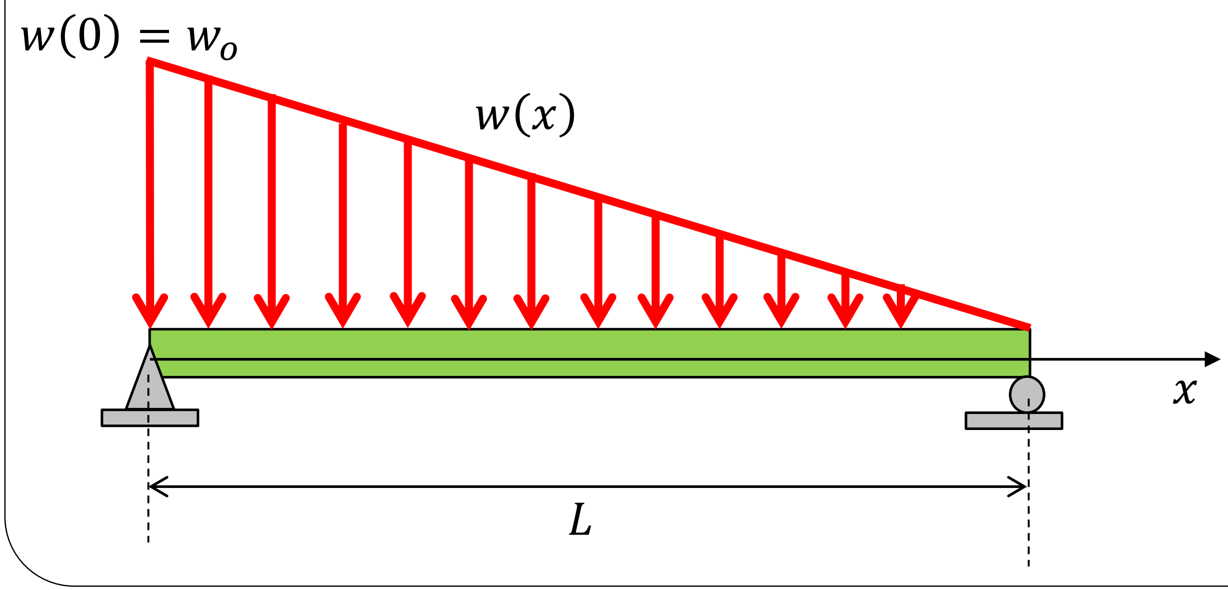
$$\bar{x} = \frac{\int_0^L w(x)\, x\, dx}{\int_0^L w(x)\, dx}$$

You will learn more detail later, but F acts through a point $x = \bar{x}$ which is called the geometric center or centroid of the area A under the loading curve w(x).

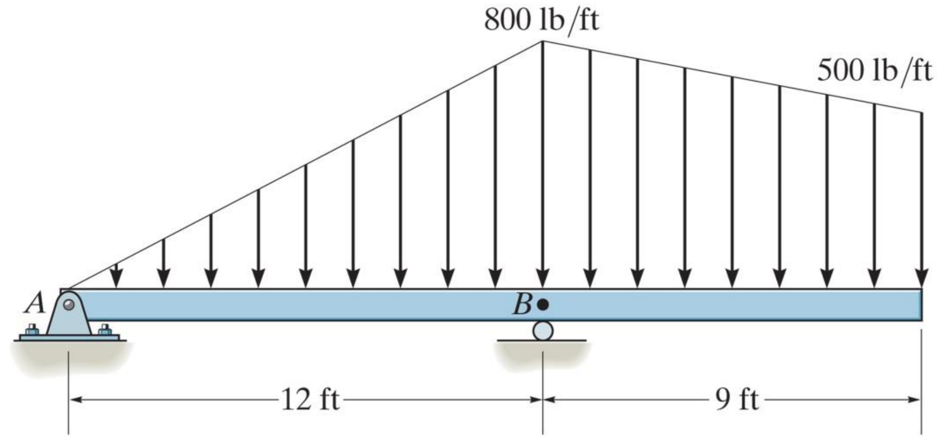
Rectangular loading



Triangular loading



Replace the loading by an equivalent resultant force and specify its location on the beam measured from point B



Replace the loading by an equivalent resultant force and specify its location on the beam measured from point A